Code No: 121AB

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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year Examinations, March/April - 2023
MATHEMATICS - I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AME, MIE, PTM)

Time: 3 hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Define Hermitian, Skew-Hermitian and Unitary Matrices. [2]
b) Define the rank and nature of a quadratic form. [3]
c) Write the geometrical interpretation of Lagrange's mean value theorem. [2]
d) Explain in brief the method of Lagrange multipliers. [3]
e) Show that $\beta(m,n) = \beta(n,m)$. [2]
f) Evaluate $\int_0^1 \int_1^2 \int_2^3 xy^2 z^3 \, dxdydz$. [3]
g) Write the differential equation of the family of orthogonal trajectories of the curves $xy = c^2$. [2]
h) Find the general solution of $y'' + 2y' + y = 0$. [3]
i) Write the linearity property in Laplace transforms. [2]
j) Define a periodic function. What is the Laplace transform of a periodic function? [3]

PART - B

(50 Marks)

2.a) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{pmatrix}$.

b) Solve completely, the system of equations $x + y - 2z + 3w = 0$; $x - 2y + z - w = 0$; $4x + y - 5z + 8w = 0$; $5x - 7y + 2z - w = 0$. [5+5]

OR

3. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence find its inverse. [10]

4. State Lagrangers mean value theorem, and using it prove that when $0<a<b<1$, $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. [10]

OR

5. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. [10]

6.a) Express the following integrals in terms of Gamma function:

i) $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta$ ii) $\int_0^{\infty} a^{-bx^2}dx$

b) Find $\Gamma\left(-\frac{11}{2}\right)$. [5+5]

OR

7.a) By changing into polar coordinates, evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$.

b) Using triple integral, find the volume of a sphere of radius 'r'. [5+5]

8.a) Solve $xy(1+xy^2)\frac{dy}{dx}=1$.

b) If the temperature of the air is 30 ^{0}C, and the substance cools from 100 ^{0}C to 80 ^{0}C in 10 minutes, find the temperature of the substance after 20 minutes. [5+5]

OR

9. Solve $(D-2)^2y=8(e^{2x}+\sin 2x+x^2)$. [10]

10. Find the Laplace transform of the function:

a) $f(t)=2e^{-3t}\sin t+3e^{t}\sin 4t+7\sqrt{t}$

b) $\int_0^{t}\frac{\sin t}{t}dt$ [5+5]

OR

11. Solve by Laplace transform technique $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t,\ y=\frac{dy}{dt}=0\ when\ t=0$. [10]

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