Code No: 121AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD B. Tech I Year Examinations, March/April - 2023 MATHEMATICS - I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AME, MIE, PTM)

Time: 3 hours Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

- ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
- iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

- 1.a) Define Hermitian, Skew-Hermitian and Unitary Matrices. [2]
 - b) Define the rank and nature of a quadratic form.
 c) Write the geometrical interpretation of Lagrange's mean value theorem.
 [2]
 - d) Explain in brief the method of Lagrange multipliers. [3]
- e) Show that $\beta(m,n) = \beta(n,m)$. [2]
- f) Evaluate $\int_{0.12}^{1.23} \int xy^2z \, dx \, dy \, dz$. [3]
- g) Write the differential equation of the family of orthogonal trajectories of the curves $xy = c^2$. [2]
- h) Find the general solution of y'' + 2y' + y = 0. [3]
- i) Write the linearity property in Laplace transforms. [2]
- j) Define a periodic function. What is the Laplace transform of a periodic function? [3]

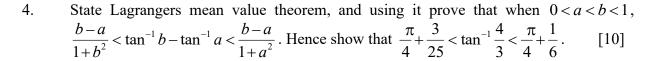
PART - B

(50 Marks)

- 2.a) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{pmatrix}$.
 - b) Solve completely, the system of equations x+y-2z+3w=0; x-2y+z-w=0; 4x+y-5z+8w=0; 5x-7y+2z-w=0. [5+5]

OR

3. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence find its inverse.



OR

- 5. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [10]
- 6.a) Express the following integrals in terms of Gamma function:

i)
$$\int_{0}^{\pi/2} \sqrt{\tan \theta} d\theta$$

ii)
$$\int_{0}^{\infty} a^{-bx^2} dx$$

b) Find
$$\Gamma\left(-\frac{11}{2}\right)$$
.

[5+5]

OR

- 7.a) By changing into polar coordinates, evaluate $\int_{0.0}^{\infty} \int_{0}^{\infty} e^{-(x^2+y^2)} dx dy$.
 - b) Using triple integral, find the volume of a sphere of radius 'r'. [5+5]
- 8.a) Solve $xy(1+xy^2)\frac{dy}{dx} = 1$.
 - b) If the temperature of the air is 30 °C, and the substance cools from 100 °C to 80 °C in 10 minutes, find the temperature of the substance after 20 minutes. [5+5]

OR

9. Solve
$$(D-2)^2 x^{1/2} 8(e^{2x} + \sin 2x + x^2)$$
. [10]

- 10. Find the Laplace transform of the function:
 - a) $f(t) = 2e^{-3t} \sin t + 3e^t \sin 4t + 7\sqrt{t}$

b)
$$\int_{0}^{t} \frac{\sin t}{t} dt$$
 [5+5]

OR

11. Solve by Laplace transform technique
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$$
, $y = \frac{dy}{dt} = 0$ when $t = 0$.

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